Cost Analysis of Proton Exchange Membrane Fuel Cell Systems

Ai-Jen Hung

Dept. of Chemical Engineering, National Taiwan University, Taipei 106-17, Taiwan

Yih-Hang Chen and Lung-Yu Sung

Energy and Environmental Lab., Industrial Technology Research Institute, Hsinchu 310, Taiwan

Cheng-Ching Yu

Dept. of Chemical Engineering, National Taiwan University, Taipei 106-17, Taiwan

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Tradeoff between capital cost and the operating cost can be seen in the design of proton exchange membrane fuel cell systems. The polarization curve indicates that operating in the region of lower current densities implies less operating cost (hydrogen fuel) and higher capital cost (larger membrane electrode assembly area). The opposite effects are observed when one operates in the region of higher current densities. Therefore, an appropriate design should take both factors into account and the optimality depends on the corresponding costs of hydrogen and membrane area. An analytical cost model is constructed to describe such an economic balance in a proton exchange membrane fuel cell system. The objective function of the optimization is the total annual cost. Six scenarios are used to illustrate the optimal design based on the total annual cost as cost and materials factors fluctuate. © 2008 American Institute of Chemical Engineers AIChE J, 54: 1798–1810, 2008

Keywords: proton exchange membrane fuel cell, cost model, tradeoff in design, optimal design

Introduction

During the design phase of chemical processes, a tradeoff between the capital costs and the operating costs can often be seen. It is important to identify the design variables which determine such an economic balance and then the optimal design can be achieved. Figure 1 shows the typical design example of a distillation column. A decrease in the reflux ratio leads to a column with a larger number of trays (higher capital cost) and a smaller heat input (lower operating cost). If a higher reflux ratio is chosen for the design, then a col-

umn becomes fewer trays (lower capital cost) and higher energy requirement (higher operating cost). Thus, the reflux ratio is a dominant design variable in the design when both capital and operating costs are considered. This concept can be extended to the design of proton exchange membrane (PEM) fuel cell systems and that is the objective of this work.

The polarization curve, which describes the response of the cell voltage to the change in the average current density, determines the characteristic of a PEM fuel cell system.³⁻⁷ The capital cost of a PEM fuel cell system includes: the membrane electrode assembly (MEA), the bipolar plates, (two most expensive components^{8,9}) and the auxiliaries such as the humidifiers, the air compressor, and the water coolant, the costs of which are strongly correlated with any variation in the MEA area. From the polarization curve, it is apparent

Correspondence concerning this article should be addressed to C.-C. Yu at ccyu@ntu.edu.tw.

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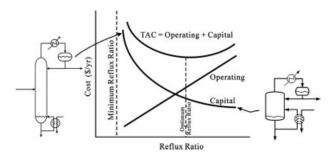


Figure 1. Tradeoff between capital cost (tray numbers) and operating cost (energy) in the design of a distillation column.

that the PEM fuel cell system is fuel (hydrogen) economical when operating in the region of low current densities, however, in practice, the system is generally operated at medium-to-high current densities, 0.6–0.7 V.¹⁰ This demonstrates the tradeoff between the operating cost and the capital investment.

In this work, the analytical expression of the cost model is derived to describe the tradeoff between the capital cost (MEA area) and the operating cost (hydrogen flow rate) for a desired power output. First, a linear approximation of a polarization curve is assumed, ^{10–13} and then the MEA area and the hydrogen flow rate can be expressed in terms of the dominant design variable, namely, the average current density. With the cost data available, one can achieve an optimal design by balancing the capital cost and the operating cost for a given power output. Next, the effects of the following factors, the MEA (and bipolar plate) cost and the hydrogen price, the reliability factor (life-cycle) for the MEA, the material factors for the MEA, on the design and the operation of PEM fuel cell systems are explored. Finally, given the projection of the MEA cost and the hydrogen price, the impact on PEM fuel cell systems are also investigated.

PEM Fuel Cell System

Operating components in hydrogen-air PEM fuel cell systems

Figure 2 shows the major operating units in a hydrogen-air PEM fuel cell system. As can be seen, pure hydrogen as a fuel at the anode is supplied and air as an oxidant at the cathode is fed by a compressor. Both streams flow through humidifiers and the exhaust reactant gases are continuous in flow-through mode. The operating cell temperature is controlled by the water coolant. As a whole, four auxiliaries, the air compressor, the anode and cathode humidifiers, and the water coolant, are needed to operate the fuel cell.

System efficiency

The system efficiency of a PEM fuel cell (η_{system}) is defined as 11,12,14 the ratio of the power output to the total energy consumption, which is the sum of the energy consumed by auxiliaries and the lower heating value of hydrogen. That is:

$$\begin{split} \eta_{\text{system}} &= \frac{P \cdot A_{\text{MEA}}}{E_{\text{total}}} = \frac{P \cdot A_{\text{MEA}}}{E_{\text{auxiliary}} + \text{LHV}_{\text{H}_2}} \\ &= \frac{P \cdot A_{\text{MEA}}}{(W_{\text{comp}} + Q_{\text{humid,a}} + Q_{\text{humid,c}} + Q_{\text{cool}}) + \text{LHV}_{\text{H}_2}} \end{split} \tag{1}$$

where P is the power density, $A_{\rm MEA}$ is the total MEA area, $E_{\rm total}$ is the total energy consumption, $E_{\rm auxiliary}$ is the energy consumed by auxiliaries, LHV $_{\rm H_2}$ is the lower heating value of total hydrogen flow, $W_{\rm comp}$ is the work done by the air compressor, $Q_{\rm humid,a}$ is the heat required by the anode humidifier, $Q_{\rm humid,c}$ is the heat required by the cathode humidifier, and $Q_{\rm cool}$ is the heat removed by the water coolant. Table 1 summarizes the equations for auxiliaries and lower heating value of hydrogen and Table 2 gives the corresponding variables for Table 1.

Once the polarization curve, Figure 3A, becomes available, the corresponding power density can be generated by applying $P = I_{\text{avg}} \cdot V$ as shown in Figure 3B. The system efficiency can be computed from Tables 1 and 2, as shown in Figure 3C. Generally, the operating cell voltage is set between 0.6 and 0.7 V. Without loss of generality, the operating cell voltage is selected at 0.65 V as an illustration. As can be seen, neither the produced power output nor the system efficiency is at their maxima. Because the system efficiency only considers the operating energy consumption but does not take account of the capital investment, it is important to construct a cost model to describe such an economic tradeoff in a PEM fuel cell system.

Economic Analysis of PEM Fuel Cell Systems

Linear approximation of a polarization curve

The polarization curve is the most important characteristic of a fuel cell and its performance. Because the practical operating range (0.6–0.7 V) comes within the ohmic loss zone as shown in Figure 4A, it is fair to use an appropriate linear polarization curve to describe the characteristics of fuel cells for our purposes. The cell voltage is assumed to be a linear function of the average current density, which can be expressed in two parameters: $V_{\rm o}$ is the intercept of the polarization curve, and $k_{\rm IV}$ is the slope of the polarization curve.

$$V(I_{\text{avg}}) = V_{\text{o}} - k_{\text{IV}} I_{\text{avg}}$$
 (2)

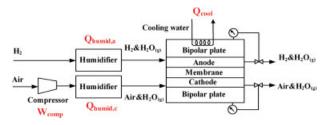


Figure 2. Hydrogen-air PEM fuel cell systems.

Table 1. Equations for Auxiliaries and Lower Heating Value of Hydrogen

	Remarks	Equations
$Q_{ m humid,a}[W]$	Heat required by the anode humidifier	$Q_{ ext{humid,a}} = M_{ ext{H}_2, ext{in}} \cdot C_{ ext{p,H}_2} \cdot (T_{ ext{humid,a}} - T_{ ext{o}}) + M_{ ext{w,a,in}}^{ ext{v}} \cdot \Delta H_{ ext{vap}}$
$Q_{\mathrm{humid,c}}[W]$	Heat required by the cathode humidifier	$Q_{ m humid,c} = M_{ m Air,in} \cdot C_{ m p,Air} \cdot (T_{ m humid,c} - T_{ m o}) + M_{ m w,c,in}^{ m v} \cdot \Delta H_{ m vap}$
$W_{\rm comp}[W]$	Work done by the air compressor	$W_{\mathrm{comp}} = M_{\mathrm{Air,in}} \cdot C_{\mathrm{p,Air}} \cdot T_{\mathrm{o}} \cdot \left[\left(\frac{P_{\mathrm{c,out}}}{P_{\mathrm{c,in}}} \right)^{\frac{1.4-1}{1.4}} - 1 \right]$
		$Q_{ m cool} = -M_{ m in}C_{ m p,in}T_{ m in} + M_{ m out}C_{ m p,out}T_{ m out} + \Delta H_{ m rxn}^{\circ} \cdot r$
		$= -(M_{\rm H_2,in}C_{\rm p,H_2}T_{\rm a,in} + M_{\rm w,a,in}^{\rm v}C_{\rm p,w}T_{\rm a,in} + M_{\rm Air,in}C_{\rm p,Air}T_{\rm c,in}$
$Q_{\mathrm{cool}}[W]$	Heat removed by cooling water	$+ M_{\mathrm{w,c,in}}^{\mathrm{v}} C_{\mathrm{p,w}} T_{\mathrm{c,in}}) + (M_{\mathrm{H_2,out}} C_{\mathrm{p,H_2}} T_{\mathrm{cell}} + M_{\mathrm{w,a,out}}^{\mathrm{v}} C_{\mathrm{p,w}} T_{\mathrm{cell}})$
		$+ M_{\mathrm{Air,out}} C_{\mathrm{p,Air}} T_{\mathrm{cell}} + M_{\mathrm{w,c,out}}^{\mathrm{v}} C_{\mathrm{p,w}} T_{\mathrm{cell}})$
		$+ \Delta H_{ m rxn}^{\circ} \cdot (M_{ m H_2,in} - M_{ m H_2,out})$
$\mathrm{LHV}_{\mathrm{H}_2}[W]$	Total lower heating value of hydrogen	$\Delta H_{ m LHV, H_2} \cdot M_{ m H_2, in}$

where V is the cell voltage, and $I_{\rm avg}$ is the average current density.

The power density is a product of the average current density and the cell voltage, which can be represented as:

$$P = I_{\text{avg}} \cdot V = V_{\text{o}} \cdot I_{\text{avg}} - k_{\text{IV}} (I_{\text{avg}})^2$$
 (3)

So the cell voltage is simply a linear function of $I_{\rm avg}$ and the power density is a quadratic function of $I_{\rm avg}$.

Nondimensionalization

Figure 4B shows the power density versus the average current density. The maximum power density also falls within the ohmic loss zone. Therefore, it is appropriate to use the peak point as the reference point for nondimensionalizing the equations.

The gradient of the power density with respect to the average current density can be derived to obtain the average current density at peak power density.

$$\left. \frac{dP}{dI_{\text{avg}}} \right|_{I_{\text{avg}} = I_{\text{avg}}^*} = 0 \tag{4}$$

The average current density corresponding to the peak power density is simply:

$$I_{\text{avg}}^* = \frac{V_{\text{o}}}{2k_{\text{IV}}} \tag{5}$$

The cell voltage corresponding to the peak power density can be expressed as:

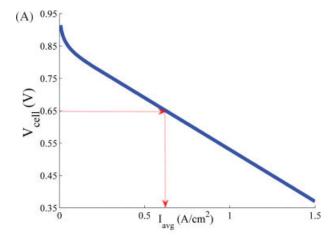
$$V^* = \frac{V_o}{2} \tag{6}$$

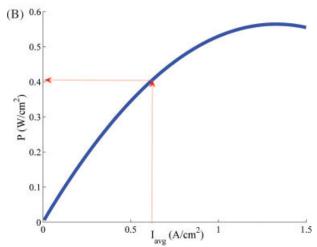
And the peak power density is:

$$P^* = V_{\rm o} \cdot I_{\rm avg}^* - k_{\rm IV} \left(I_{\rm avg}^* \right)^2 = \frac{V_{\rm o}^2}{4k_{\rm IV}}$$
 (7)

Table 2. Corresponding Variables in Table 1

	Remarks	Equations
$M_{\rm H_2,in}~({\rm mol/s})$	Molar flow rate of hydrogen at the anode input	$M_{ m H_2,in} = \left(rac{I_{ m avg}}{2\cdot F}\cdot A_{ m MEA} ight)\cdot S_{ m H_2}$
$M_{\mathrm{w,a,in}}^{\mathrm{v}} \; (\mathrm{mol/s})$	Molar flow rate of water vapor at the anode input	$M_{ ext{w,a,in}}^{ ext{v}} = M_{ ext{H}_2, ext{in}} \cdot \left(rac{P_{ ext{wa}}^{ ext{rad}}(T_{ ext{humid,a}})}{P_{ ext{a}} - P_{ ext{wa}}^{ ext{sat}}(T_{ ext{humid,a}})} ight)$
$M_{\rm Air,in}~({ m mol/s})$	Molar flow rate of air at the cathode input	$M_{ m Air,in} = \left(rac{I_{ m avg}}{4 \cdot F} \cdot A_{ m MEA} ight) \cdot S_{ m O_2} \cdot rac{1}{0.21}$
$M_{\mathrm{w,c,in}}^{\mathrm{v}} \; (\mathrm{mol/s})$	Molar flow rate of water vapor at the cathode input	$M_{ ext{w,c,in}}^{ ext{v}} = M_{ ext{Air,in}} \left(rac{P_{ ext{w,c}}^{ ext{std}}(T_{ ext{humid,c}})}{P_c - P_{ ext{w,c}}^{ ext{w,c}}(T_{ ext{humid,c}})} ight)$
$M_{\rm H_2,out} \ (\rm mol/s)$	Molar flow rate of hydrogen at the anode output	$M_{\mathrm{H_2,out}} = (S_{\mathrm{H_2}} - 1) \cdot \left(\frac{l_{\mathrm{avg}}}{2 \cdot F} \cdot A_{\mathrm{MEA}}\right)$
$M_{\mathrm{w,a,out}}^{\mathrm{v}}\left(\mathrm{mol/s}\right)$	Molar flow rate of water vapor at the anode output	$M_{ ext{w,a,out}}^{ ext{v}} = M_{ ext{H}_2, ext{out}} \cdot \left(rac{P_{ ext{w,a}}^{ ext{t}}(T_{ ext{cell}})}{P_a - P_{ ext{w,a}}^{ ext{w}}(T_{ ext{cell}})} ight)$
$M_{\rm Air,out}~({ m mol/s})$	Molar flow rate of air at the cathode output	$M_{\mathrm{Air,out}} = (S_{\mathrm{O_2}} - 1) \cdot \left(\frac{I_{\mathrm{avg}}}{4F} \cdot A_{\mathrm{MEA}} \right) \cdot \frac{1}{0.21}$
$M_{\rm w,c,out}^{\rm v}~({\rm mol/s})$	Molar flow rate of water vapor at the cathode output	$M_{ ext{w,c,out}}^{ ext{v}} = M_{ ext{Air,out}} \cdot \left(\frac{P_{ ext{w.c}}^{ ext{va.c}}(T_{ ext{cell}})}{P_c - P_{ ext{w.c}}^{ ext{va.c}}(T_{ ext{cell}})} \right)$
$C_{\mathrm{p,H_2}}\left(\mathrm{J/mol}\mathrm{K}\right)$	Heat capacity of hydrogen	$C_{\text{p,H}_2} = R \cdot \left(3.429 + 0.422 \times 10^{-3} \cdot T + \frac{0.083 \times 10^5}{T^2} \right)$
$C_{p,Air}\left(\mathrm{J/mol}\mathrm{K}\right)$	Heat capacity of air	$C_{\text{p,Air}} = R \cdot \left(3.355 + 0.575 \times 10^{-3} \cdot T - \frac{0.016 \times 10^5}{T^2}\right)$
$C_{p, w} (J/\text{mol } K)$	Heat capacity of air	$C_{\text{p,w}} = R \cdot \left(3.47 + 1.45 \times 10^{-3} \cdot T + \frac{0.121 \times 10^5}{T^2} \right)$
$\Delta H_{\mathrm{vap}} \; (\mathrm{J/mol})$	Enthalpy of water evaporation	$\Delta H_{\text{vap}} = 45070 - 41.94T + 3.44 \times 10^{-3}T^2 + 2.548 \times 10^{-6}T^3 - 8.98 \times 10^{-10}T^4$
$P_{\rm w}^{\rm sat}$ (atm)	Saturation pressure of water vapor	$P_{\rm w}^{\rm sat} = 10^{-2.18 + 2.9 \times 10^{-2} T - 9.18 \times 10^{-5} T^2 + 1.44 \times 10^{-7} T^3}$





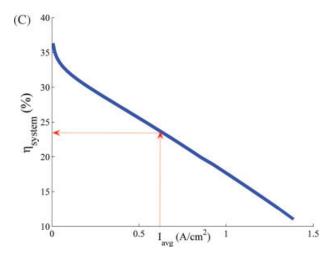


Figure 3. (A) Cell voltage vs. the average current density, (B) power density versus average current density, (C) system efficiency vs. the average current density; dashed line indicating typical cell working voltage of 0.65 V.

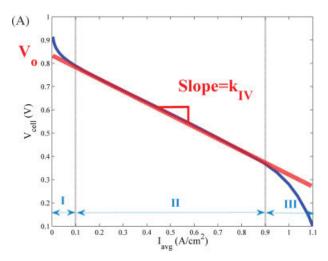
Relationship between MEA area and hydrogen flow rate

For a desired power output ($P_{\rm des}$), the product of the peak power density and the total MEA area at peak power density is equal to the product of the power density and the total MEA area for any given $I_{\rm avg}$.

$$P_{\text{des}} = P^* \cdot A_{\text{MEA}}^* = P \cdot A_{\text{MEA}} \tag{8}$$

Substituting P^* in Eq. 7 into Eq. 8, we have:

$$P_{\text{des}} = \left[\frac{V_{\text{o}}^2}{4k_{\text{IV}}} \right] \cdot A_{\text{MEA}}^* \tag{9}$$



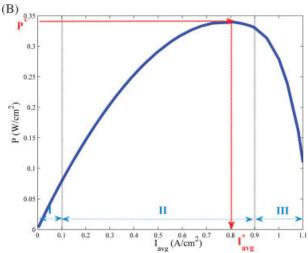


Figure 4. (A) Linear approximation of a fuel cell polarization curve, with three zones of losses over the cell cross-section: I: activation loss zone, II: ohmic loss zone, III: concentration loss zone and (B) the power density vs. the average current density with the maximum power density P^* and corresponding average current density $I^*_{\rm avg}$.

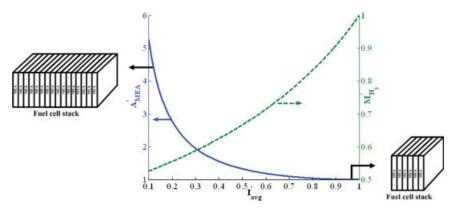


Figure 5. Tradeoff between the dimensionless MEA area (capital cost) and the dimensionless hydrogen flow rate (operating cost) as the dimensionless average current density varies from 0.1 to 1 for a given power output.

Similarly, for any given I_{avg} , the desired power output can be obtained by substituting Eq. 3 into Eq. 8.

$$P_{\text{des}} = \left[V_{\text{o}} \cdot I_{\text{avg}} - k_{\text{IV}} (I_{\text{avg}})^2 \right] \cdot A_{\text{MEA}}$$
 (10)

Dividing Eq. 10 by Eq. 9, one obtains:

$$\frac{A_{\text{MEA}}}{A_{\text{MEA}}^*} = \frac{V_{\text{o}} \cdot I_{\text{avg}} - k_{\text{IV}} (I_{\text{avg}})^2}{V_{\text{o}}^2 / 4k_{\text{IV}}}$$

$$= 2 \cdot \left(\frac{2k_{\text{IV}}}{V_{\text{o}}}\right) \cdot I_{\text{avg}} - \left(\frac{2k_{\text{IV}}}{V_{\text{o}}} \cdot I_{\text{avg}}\right)^2 \tag{11}$$

Substituting I_{avg}^* for $V_{\text{o}}/2k_{\text{IV}}$, we have:

$$\left(\frac{A_{\text{MEA}}}{A_{\text{MEA}}^*}\right) = \left(\frac{I_{\text{avg}}}{I_{\text{avg}}^*}\right)^{-1} \cdot \left[2 - \left(\frac{I_{\text{avg}}}{I_{\text{avg}}^*}\right)\right]^{-1}$$
(12)

The dimensionless MEA area ($A'_{\rm MEA} = A_{\rm MEA}/A^*_{\rm MEA}$) is a function of the dimensionless average current density ($I'_{\rm avg} =$ $I_{\text{avg}}/I_{\text{avg}}^*$) while the range of the dimensionless average current density of interest is between 0 and 1. That is:

$$A'_{\text{MEA}} = (I'_{\text{avg}})^{-1} (2 - I'_{\text{avg}})^{-1} \quad 0 < I'_{\text{avg}} < 1$$
 (13)

where A'_{MEA} is the ratio of the total MEA area to the total MEA area at peak power density for any given $I'_{avg} \cdot I'_{avg}$ is the ratio of the average current density to the average current density at peak power density.

The hydrogen flow rate at peak power density is directly proportional to the average current density at peak power density (according to Faraday's Law).

$$M_{\rm H_2}^* = \left(\frac{I_{\rm avg}^*}{2F} \cdot A_{\rm MEA}^*\right) \cdot S_{\rm H_2} \tag{14}$$

where $S_{\rm H_2}$ is the stoichiometric ratio of hydrogen and F is Faraday's constant.

Similarly, the hydrogen flow rate for any given I_{avg} can be expressed as:

$$M_{\rm H_2} = \left(\frac{I_{\rm avg}}{2F} \cdot A_{\rm MEA}\right) \cdot S_{\rm H_2} \tag{15}$$

Dividing Eq. 15 by Eq. 14, it gives:

$$\left(\frac{M_{\rm H_2}}{M_{\rm H_2}^*}\right) = \left(\frac{I_{\rm avg}}{I_{\rm avg}^*}\right) \cdot \left(\frac{A_{\rm MEA}}{A_{\rm MEA}^*}\right)$$
(16)

Substituting $(A_{\text{MEA}}/A_{\text{MEA}}^*)$ of Eq. 12 into Eq. 16, we have:

$$\left(\frac{M_{\rm H_2}}{M_{\rm H_2}^*}\right) = \left(2 - \frac{I_{\rm avg}}{I_{\rm avg}^*}\right)^{-1}$$
(17)

The dimensionless hydrogen molar flow rate $(M'_{H_2} = M_{H_2})$ $M_{\rm H_2}^*$) is a function of dimensionless average current density while the range of the dimensionless current density is between 0 and 1. That is:

$$M'_{\rm H_2} = (2 - I'_{\rm avg})^{-1} \quad 0 < I'_{\rm avg} < 1$$
 (18)

where $M'_{\rm H_2}$ is the dimensionless hydrogen flow rate for any given I'_{avg} .

Figure 5 shows that the dimensionless MEA area and the dimensionless hydrogen flow rate change as a function of dimensionless average current density (Eqs. 13 and 18). For a given power output, there is an economic tradeoff between the MEA area (capital cost) and the hydrogen flow rate (operating cost) as the current density varies. Therefore, the dimensionless average current density is a dominant design variable for a PEM fuel cell system. The capital costs can be reduced by using a smaller number of cells (MEA area), but, in doing this, the operating cost goes up as a result of an increase in the hydrogen flow.

Cost Model

Total annual cost (TAC)

The TAC is the objective function used to evaluate fuel cell designs. The definition of TAC is:

$$TAC = \frac{\text{capital cost}}{\text{MEA life-cycle}} + \text{operating cost}$$
 (19)

A 5 kW PEM fuel cell system is used to illustrate the tradeoff in design (Figure 6). Table 3 summarizes the nomi-

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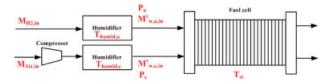


Figure 6. 5 kW PEM fuel cell system.

nal design and corresponding operating conditions for this 5 kW system. Note that the nominal design is based on the typical operating cell voltage of 0.65 V, and the stoichiometric ratios are set to 1.5 and 2.5 for hydrogen and oxygen, respectively. From the polarization curve, Figure 7, a linear approximation is made to obtain two model parameters, $V_{\rm o}$ and $k_{\rm IV}$. Once the model parameter becomes available, one is ready to compute the following information: the total MEA area corresponding to peak power density $(A_{\rm avg}^*)$, the average current density corresponding to the peak power density $(I_{\rm avg}')$, the peak power density (P^*) , the cell voltage corresponding to peak power density (V^*) , and the hydrogen flow rate corresponding to peak power density $(M_{\rm H_{*}}^*)$, which are shown in Table 4.

Capital cost

Two of the most expensive components in a fuel cell stack are the MEA and the bipolar plates. The active area of the bipolar plate, which contains the flow channel, is the same as the MEA area, and the cost is also proportional to the MEA area. Thus, it is reasonable to use simply the total MEA area, instead of two separate areas, to correlate the cost of these two components. So, we have:

Capital cost
$$\propto A_{\text{MEA}}$$
 (20)

In a fuel cell stack, the total MEA area is the product of the number of cells and the individual MEA area:

$$A_{\text{MEA}} = n_{\text{cell}} \cdot a_{\text{MEA}} \tag{21}$$

where n_{cell} is the number of cells and a_{MEA} is the individual cell MEA area, which generally falls between 25 and 780 cm².^{7,10,11} Because a constraint exists in the individual MEA area (a_{MEA}), there are two ways to handle total MEA area

Table 3. Nominal Design of the 5 kW PEM Fuel Cell System

	Remark	Value
P_{des}	Desired stack power output	5 kW
$n_{\rm cell}$	Number of cells	53
a_{MEA}	Single MEA area	225 cm^2
A_{MEA}	Total MEA area	11925 cm ²
$T_{\rm st}$	Stack temperature	60°C
$T_{\text{humid,a}}$	Anode humidification temperature	65°C
$T_{ m humid,c}$	Cathode humidification temperature	65°C
$M_{\mathrm{H_2,in}}$	Hydrogen inlet flow rate	0.0607 mol/s
$M_{ m Air,in}$	Air inlet flow rate	0.2409 mol/s
$M_{\text{w.a.in}}^{\text{v}}$	Water vapor inlet flow rate at anode	0.0118 mol/s
$M_{\mathrm{w,a,in}}^{\mathrm{v}}$ $M_{\mathrm{w,c,in}}^{\mathrm{v}}$	Water vapor inlet flow rate at cathode	0.0470 mol/s
$S_{\rm H_2}^{\rm w,c,m}$	Hydrogen stoichiometric ratio	1.5X
$S_{\mathcal{O}_2}$	Oxygen stoichiometric ratio	2.5X
$P_{\rm a}^{\circ_2}$	Anode pressure	1.5 atm
$P_{\rm c}^{"}$	Cathode pressure	1.5 atm

 $(A_{\rm MEA})$ variation. One is to change the number of cells and this is denoted as "numbering-up." The other is to change the membrane size, called "scaling-up" hereafter. Thus, the capital cost can be expressed as:

Capital cost =
$$w_1 \cdot n_{\text{cell}} \cdot a_{\text{MFA}}^n$$
 (22)

where w_1 is a cost weighting factor which represents the cost of the MEA and bipolar plate per square centimeter and n is the exponent to account for area variation for an individual cell. A change in the number of cells to achieve the desired power output indicates that we are varying n_{cell} with the exponent of n=1. As for the individual membrane size variation, a regression of the cost vs. area gives an exponent of 0.62 (see the Appendix for the derivation). So, Eq. 22 gives a general form for handling the total MEA area variation.

Substituting Eq. 12 into Eq. 22, the capital cost can be expressed in terms of the dimensionless average current density.

Capital cost =
$$w'_1 \cdot \left[2I'_{avg} - \left(I'_{avg} \right)^2 \right]^{-n}$$
 (23)

where

$$w_1' = \left[w_1 \cdot n_{\text{cell}} \cdot \left(\frac{P_{\text{des}}}{n_{\text{cell}} \cdot (V_o^2 / 4k_{\text{IV}})} \right)^n \right]$$

Applying the price of 2007 for the MEA area (Table 5), the capital cost of the nominal design of the 5 kW PEM fuel cell system can be computed from Eq. 23. It gives:

Capital cost =
$$\left[0.3623 \cdot 53 \cdot \left(\frac{5000}{53 \cdot (0.84^2/4 \cdot 0.29)} \right) \right]$$

$$\times \left[2 \cdot 0.4523 - (0.4523)^2 \right]^{-1} = \$4,254$$

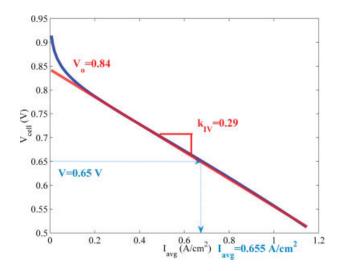


Figure 7. Linear approximation of the polarization curve with $V_{\rm o}=0.84$ and $k_{\rm IV}=0.29$ and the operating cell voltage is set to 0.65 V as indicated by the dashed line.

Table 4. Computed Variables with Respect to the Peak Power Density and Dimensionless Average Current Density of the 5 kW PEM Fuel Cell System

	Remark	Value
A*MEA I*avg I'avg P* V*	Total MEA area corresponding to the peak power density Average current density corresponding to the peak power density Dimensionless average current density Peak power density	8220 cm ² 1.448 A/cm ² 0.4523 0.608 W/cm ²
$M_{ m H_2}^*$	Cell voltage corresponding to the peak power density Hydrogen flow rate corresponding to the peak power density	0.42 V 0.093 mol/s

Operating cost

From the energy balance, one can compute the energy consumption for all components in the fuel cell system from Tables 1 and 2.⁷ Results show that almost 90% of the total energy consumption comes from the fuel, hydrogen, as shown in Figure 8. The remainder comes from the auxiliaries. Hence, it is reasonable to assume that the lower heating value of the hydrogen, or simply the hydrogen flow rate, dominates the operating cost. In other words, we have:

Operating cost
$$\propto M_{\rm H_2}$$
 (24)

The operating time of 8760 hr/yr is used and then the operating cost can be represented as:

Operating cost =
$$\left(w_2 \frac{\$}{\text{mol}}\right) \cdot \left(M_{\text{H}_2} \frac{\text{mol}}{\text{s}}\right) \cdot \left(3600 \frac{\text{s}}{\text{hr}}\right)$$

 $\times \left(8760 \frac{\text{hr}}{\text{yr}}\right)$ (25)

where w_2 is the unit price of hydrogen (\$/mol). Combining Eqs. 18 and 25, the operating cost can be represented in terms of the dimensionless average current density:

Operating cost =
$$w_2' \cdot \left[2 - I_{\text{avg}}'\right]^{-1}$$
 (26)

where

$$w_2' = \left[w_2 \cdot \left(\frac{P_{\text{des}} \cdot S_{\text{H}_2}}{F \cdot V_{\text{o}}} \right) \cdot 3600 \cdot 8760 \right]$$

Provided with the current hydrogen unit price (Table 5) and the nominal operating conditions, the operating cost for the 5 kW PEM fuel cell system can be computed directly.

Operating cost =
$$\left[0.006 \cdot \left(\frac{5000 \cdot 1.5}{96485 \cdot 0.84} \right) \cdot 3600 \cdot 8760 \right]$$
$$\times \left[2 - 0.4523 \right]^{-1} = \$11,313$$

Table 5. Cost Data for 2007 and 2015

Cost Factor	Remark	2007	2015	Unit
w_1	Cost of MEA plus	0.3573	0.0181	\$/cm ²
w_2 w_3	bipolar plate ⁸ Hydrogen cost ¹⁵ MEA Life-cycle ¹⁶	0.006 0.23	0.004 0.57	\$/mol year

Analytical expression of total annual cost (TAC)

With the expressions of capital cost (Eq. 23) and operating cost (Eq. 26), the TAC of a PEM fuel cell can be expressed explicitly in terms of the dimensionless average current density along with weighting factors on the cost and reliability. That is:

$$TAC = \frac{w'_1}{w_3} \cdot \left[2I'_{avg} - \left(I'_{avg} \right)^2 \right]^{-n} + w'_2 \cdot \left[2 - I'_{avg} \right]^{-1}$$
 (27)

where w_3 is the MEA life-cycle. Again, for our 5 kW PEM fuel cell example, the total annual cost becomes:

$$TAC = \frac{4254}{0.23} + 11313 = \$29,810$$

When varying the dimensionless average current density, an economic tradeoff between the capital costs and the operating costs is apparent as shown in Figure 9A. The capital cost initially drops sharply in the region of low current densities and then levels off. On the other hand, the operating cost increases steadily as $I'_{\rm avg}$ increases. Corresponding MEA area is expressed in terms of number of cell (15 cm \times 15 cm) which is shown in Figure 9B. The design which

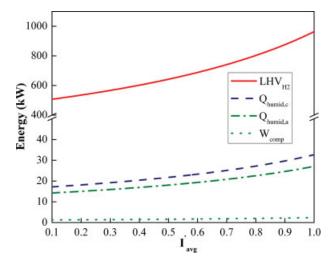
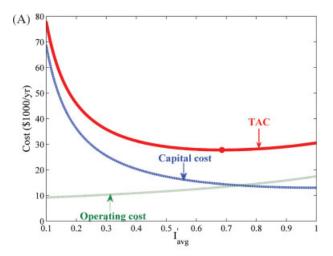


Figure 8. Energy consumption for the fuel and auxiliaries within the range of 0.1–1 of the dimensionless average current density for a given power output.



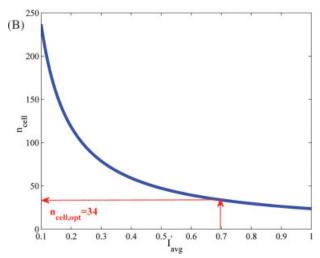


Figure 9. (A) Total annual cost (TAC), capital cost, and operating cost vs. the dimensionless average current density and (B) number of cells (15 cm × 15 cm) vs. the dimensionless average current density; optimal TAC-based design corresponding to the number of cells of 34.

gives the lowest TAC requires 34 cells, as opposed to 53 in the nominal design. A TAC saving of \$3,000 can be achieved.

Simplified system efficiency

As mentioned earlier, the majority of the total energy consumption to operate the system comes from the lower heating value of hydrogen. It is reasonable to neglect the energy consumed by auxiliaries in the system efficiency approximation. With this assumption, an analytical expression for the efficiency becomes:

$$\eta_{\text{system}} = \frac{P \cdot A_{\text{MEA}}}{E_{\text{auxiliary}} + \text{LHV}_{\text{H}_2}} \approx \frac{P \cdot A_{\text{MEA}}}{\text{LHV}_{\text{H}_2}}$$
(28)

Substituting Eqs. 3 and 15 into Eq. 28, a linear relationship between the system efficiency and the dimensionless average current density is found.

$$\eta_{\text{system}} = c \cdot \left(2 - I_{\text{avg}}'\right)$$
(29)

with

$$c = \frac{F \cdot V_0}{S_{\text{H}_2} \cdot \Delta H_{\text{LHV,H}_2}} \tag{30}$$

where $\Delta H_{\rm LHV,H2}$ (J/mol) is the lower heating value of hydrogen. Figure 10 shows the system efficiency decreases linearly with an increase in the dimensionless average current density and the TAC optimal design gives an efficiency of 29.5%.

Analytical expressions for TAC optimal design with numbering up (n = 1)

If the area of individual cell is fixed, the total MEA area variation is handled by changing the number of cells, numbering-up. In this case, we have n=1 and the expression for TAC becomes:

$$TAC = \frac{w_1''}{w_3} \cdot \left[2I_{avg}' - \left(I_{avg}'\right)^2 \right]^{-1} + w_2' \cdot \left[2 - I_{avg}' \right]^{-1}$$
 (31)

where

$$w_1'' = \left[w_1 \cdot \frac{P_{\text{des}}}{(V_o^2/4k_{\text{IV}})} \right],$$

$$w_2' = \left[w_2 \cdot \frac{P_{\text{des}} \cdot S_{\text{H}_2}}{F \cdot V_o} \cdot 3600 \cdot 8760 \right],$$

$$w_3 = \left[\text{MEA life-cycle} \right]$$

Taking the derivative of TAC with respect to the dimensionless average current density, the optimal I'_{avg} can be found by solving the following equation.

$$\left. \frac{d\text{TAC}}{dl'_{\text{avg}}} \right|_{l'_{\text{avg}} = l'_{\text{avg, opt}}} = 0 \tag{32}$$

A further simplification leads to a simple expression for the optimal average current density. It is a function of the combination of the cost and reliability factors, ξ .

$$I'_{\text{avg, opt}} = \xi \left(\sqrt{1 + \frac{2}{\xi}} - 1 \right) \tag{33}$$

where $\xi = w_1''/(w_3 \cdot w_2')$.

The optimal TAC can also be derived analytically.

$$TAC_{\text{opt}} = \frac{w_2'}{2} \cdot \left(1 + \xi - \xi \sqrt{1 + \frac{2}{\xi}}\right)^{-1}$$
 (34)

The optimal system efficiency is:

$$\eta_{\text{system, opt}} = c \cdot \left(2 + \xi - \xi \sqrt{1 + \frac{2}{\xi}}\right)$$
(35)

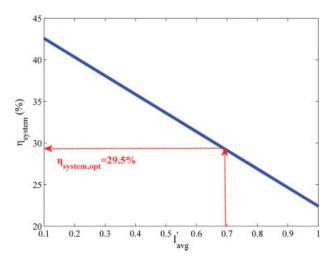


Figure 10. System efficiency vs. the dimensionless average current density for the 5 kW PEM fuel cell example with an optimal system efficiency of 29.5%.

with the constant c of Eq. 30. Equations 33 to 35 reveal that, given the unit price for the MEA (w_1) , hydrogen (w_2) , and the cycle-life of membrane (w_3) , in year, the optimized design can readily be obtained.

Analysis and Discussions

With the simple analytical expressions available, it will be interesting to see how technology and price factors affect the optimal design of PEM fuel cell. Using the predicted trend in the MEA and hydrogen prices, one will be able to see the trend of TAC optimal designs.

Cost factors: prices of MEA (and bipolar plate) and hydrogen

Because of the depletion of fossil fuels, PEM fuel cells play an important role in the renewable energy field. To make fuel cells cost-competitive with gasoline internal combustion engines, the membrane cost per square centimeter and the hydrogen cost must be reduced in the near future. Table 4 gives the cost targets in 2015.

It is important to see how the optimal TAC in a PEM fuel cell system will change in the future. Left-hand-side graphs of Figure 11 show the effects of the MEA (and bipolar plate) unit price on the dimensionless average current density $(I'_{avg,opt})$, the capital cost, the operating cost, the TAC (TAC_{opt}) , and the achievable system efficiency (η_{system}) , from TAC optimal design. The x-axis indicates the current and expected MEA unit price starting from 2007 (right; indicated by solid circles) to 2015 (left; indicated with open circles) taken from Table 5. A reduction in the unit price of MEA (and bipolar plate) results in a significant decrease in the capital cost, as compared to the operating cost. It is clear that a cost down for the MEA (and bipolar plate) unit price results in a decrease in the operating average current density and, subsequently, leads to an improved system efficiency. Conversely a reduction in the hydrogen cost causes a greater reduction in the operating cost, as compared to the capital cost, as shown in the middle graphs of Figure 11. If the hydrogen price goes down, the operating average current

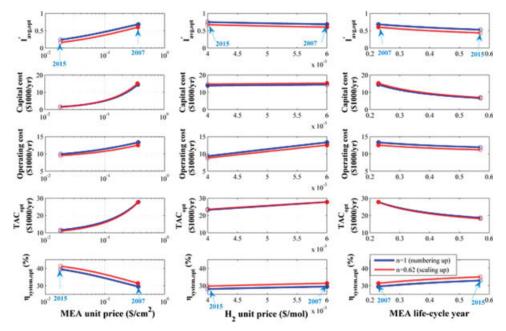


Figure 11. Cost (MEA and hydrogen) and product reliability (MEA life cycle) factors to the TAC-based optimal design and corresponding operating conditions (I_{avg}^* , capital cost, operating cost, TAC, and system efficiency) when MEA area variation handled by adding more cells (numbering up; solid line) and by enlarging single cell area (scaling up; dashed line).

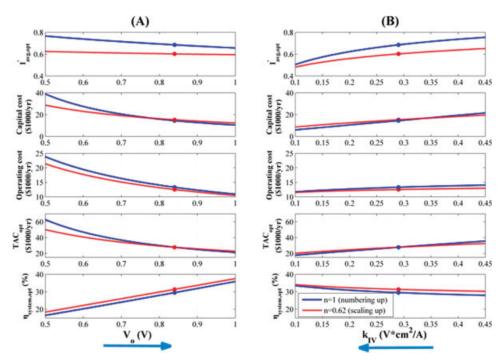


Figure 12. Material factors (varying intercept V_0 and changing slope $k_{\rm IV}$) to the TAC-based optimal design and corresponding operating conditions ($I_{
m avg}^{\prime}$, capital cost, operating cost, TAC, and system efficiency) when MEA area variation handled by adding more cells (numbering up; solid line) and by enlarging single cell area (scaling up; dashed line).

density will naturally increase. This results in less efficient PEM systems. Notice that the arguments are based on designing the optimal TAC in fuel cell systems.

Reliability factor (life-cycle) for the MEA

The heart of a PEM fuel cell system is the MEA, where the electrochemical reaction takes place to generate power output. The durability and reliability of the MEA has been improved over the past few years, and will be further improved in the future. The effects of an increase in the MEA life time (called life-cycle here) on the optimal design of a PEM fuel cell system are explored in Figure 11 (right graphs). An increase in the life-cycle for the MEA causes a greater decrease in the capital cost, as compared to the operating cost. As the MEA life-cycle increases, the optimal operating average current density decreases which results in a higher system efficiency.

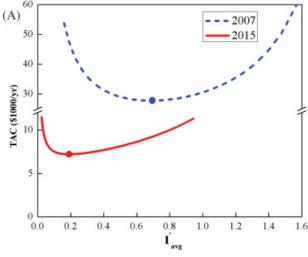
As for the TAC reduction of an optimized design, a cost down in the MEA unit price is the most effective approach, followed by the MEA durability (life-cycle), and finally the hydrogen price. Up to now, we assume the polarization curve $(V_o \text{ and } k_{IV})$ remains the same. It will be interesting to see the impact of the improvement in the materials on the optimal design of a PEM fuel cell system.

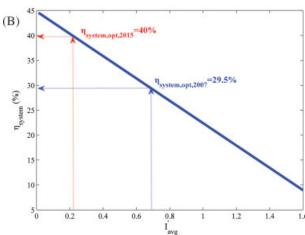
Material factors for the MEA

With the linear approximation of an polarization curve, the MEA material factors are characterized by V_0 , the intercept of the polarization curve, and k_{IV} , the slope of the polarization curve. The intercept (V_0) is the achievable MEA potential and the slope (k_{IV}) is the internal resistance of the MEA. The typical range for the intercept (V_0) is 0.5-1 V and for the slope $(k_{\rm IV})$ ranges from 0.1 to 0.45 V cm²/A.^{17–19} As can be seen in Figure 12A, the optimal TAC decreases with an increase in the intercept (V_o) . An increase in the intercept (V_o) brings about a higher system efficiency. As the slope (k_{IV}) decreases, the optimal TAC reduces as shown in Figure 12B. A decrease in the slope (k_{IV}) causes a decrease in the average current density which, subsequently, leads to a higher system efficiency. In summary, a polarization curve with a larger intercept and less internal resistance (smaller $k_{\rm IV}$) decrease can improve the system efficiency and result in a lower TAC.

Cost factors in the future

Figure 13A shows how the TAC-based optimal design varies as the projected cost data of 2015 is used. First of all, as compared to the cost in 2007, the TAC drops from \$27,800 to \$7200, 26% of the present cost. As the operating average current density decreases, the system efficiency will improve to 40%, a 10% increase from the 2007 design, as shown in Figure 13B. However, the optimal number of cells increases from 34 to 109, as shown in Figure 13C. This can be a problem for TAC-based design, if a size constraint is in effect. Nonetheless, a more efficient





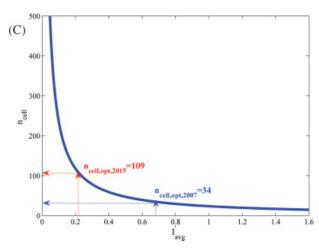


Figure 13. TAC-based optimal design based on US DOE cost scenario for 2007 and 2015.

(A) Total annual cost (TAC), (B) system efficiency, and (C) total number of cells as a result of numbering up. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

fuel cell system is expected as we see continuing drops in the MEA prices, hydrogen cost and improved membrane durability.

Two methods of varying the MEA area to achieve the same desired power output

To achieve the same power output, there are two ways to vary the MEA area: One is to change the number of cells (numbering-up, n = 1) and the other is to change the membrane size (scaling-up, n = 0.62). Figure 11 also compares five different properties for these two scenarios, numberingup (solid) versus scaling-up (dashed). An increase in the membrane size, rather than an increase in the number of cells, results in improved system efficiency when the MEA (and bipolar plate) cost decreases. When the hydrogen cost decreases, an increase in the number of cells is inferior in system efficiency, as compared to an increase in the membrane size. As the MEA life-cycle increases, an increase in the membrane size gives higher system efficiency than an increase in the number of cells causes. For a given desired power output, an increase in the membrane size is favored as compared to an increase in cell number.

Conclusions

In this work, the design tradeoff in PEM fuel cell systems has been explored. In the design of a PEM fuel cell system, the average current density plays a role similar to that of the reflux ratio in the design of distillation columns. Design at a small current density requires a large MEA area (capital cost) and less hydrogen fuel (operating cost) and the opposite is observed if we choose a larger current density. With the following assumptions: (1) linear approximation of the polarization curve and (2) the hydrogen fuel being the dominating operating cost, analytical expressions for the capital and operating costs can be derived. For total annual cost (TAC) based design, optimal design can be obtained and it transpires the optimal current density is simply a function of cost factors (unit prices of MEA and hydrogen) and material durability (membrane life-cycle). With analytical expressions for TAC-based design available, the prediction of future MEA, hydrogen prices, and materials durability can lead to a design trend projection. For the prices and material durability factors, these scenarios all result in improved system efficiency with lower TAC. Similar observations can be seen for improvement in membrane materials.

Acknowledgment

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Notation

```
A_{\text{MEA}} = \text{total MEA active area (cm}^2)
a_{\text{MEA}} = \text{single MEA active area (cm}^2)
   C_p = heat capacity (J/mol K)
     E = \text{energy (J/mol)}
     F = Faraday's constant (96,485 C/mol)
      I = \text{current density } (A/\text{cm}^2)
   k_{\text{IV}} = \text{slope of the polarization curve (V cm}^2/\text{A})
LHV = lower heating value (W)
    M = \text{molar flow rate (mol/s)}
  n_{\rm cell} = number of cells
     N = \cos t exponent
     Q = \text{heat input (W)}
```

r = reaction rate (mol/s)
R = gas constant (8.314 J/mol K)
P = power density (W/cm²)
P_{des} = desired power output (W)
W = work (W)
w₁ = sum cost of individual MEA and bipolar plate per square centimeter (\$/cm²)
w₂ = cost of hydrogen per mole (\$/mol)
w₃ = MEA life-cycle
S = stoichiometric ratio
T = temperature (K)
V = cell voltage (V)
V_o = intercept of the polarization curve (V)

Greek letters

 $\eta_{\rm system} = {
m system}$ efficiency $\Delta H_{\rm rxn}^{\rm o} = {
m standard}$ enthalpy of overall reaction (-214,818 J/mol) $\Delta H_{\rm LHV,H_2} = {
m lower}$ heating value of hydrogen (241,000 J/mol) $\Delta H_{\rm vap} = {
m heat}$ of vaporization for water (J/mol)

Subscripts

a = anode
avg = average
c = cathode
cool = coolant
comp = compressor
humid = humidifier
H₂ = hydrogen
in = inlet
O₂ = oxygen
o = standard state at 25°C
out = outlet
opt = optimal
w = water
st = stack

Superscripts

sat = saturationv = vapor

* = reference state for nondimensionalization

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Appendix

Marshall and Swift index of 2006 (\$1302.3) is used here.

(1) Cost of membrane electrode assembly (MEA) and bipolar plate in \$

Size of Membrane Electrode Assembly (MEA) and Bipolar Plate (cm)	Area (cm ²)	Cost (\$)
5 × 5	25	64
10 × 10	100	151

$$\Rightarrow C_{(\text{MEA+Bipolar})} = \left(\frac{C_{\text{MS}}}{280}\right) (0.35) \left(n_{\text{cell}}\right) \left(a_{\text{MEA}}\right)^{0.62}$$

This is the regression result using PRIMEATM 5621 (thickness 35 μ m) of Gore Inc. As for a thinner membrane, PRIMEATM 5641 (thickness 25 μ m), the exponent turns out to be 0.65. So an exponent between 0.6 and 0.7 will be a reasonable choice for general applications. The user can also do the regression for a specific membrane they are interested in.

(2) Cost of humidifier in \$

Humidifier Type	Area (cm ²)	Cost (\$)
FC-125-240-7PP	1362	1,151.5
FC-150-480-7PP	2724	1,636.4
FC-200-780-7PP	4427	2,272.7
FC-400-2500-7PP	14,188	3,606.1
FC-600-7000-7PP	39,726	6,151.5

$$\Rightarrow C_{\text{humid}} = \left(\frac{C_{\text{MS}}}{280}\right) (7.06) \left(A_{\text{humid}}\right)^{0.494}$$

where $A_{\rm humid}[{\rm m^2}] = \frac{Q_{\rm humid}}{U \cdot \Delta T}$, $U = 788.45 \frac{{\rm W}}{{\rm m^2 \cdot K}}$, $\Delta T = 10{\rm K}$

(3) Cost of compressor in \$

$$\Rightarrow C_{\text{comp}} = \left(\frac{C_{\text{MS}}}{280}\right) (517.5) \left(W_{\text{comp}}\right)^{0.82} F_{\text{d}}$$

Because the costs of humidifier and compressor are much less than that of the MEA, theses two costs are not taken into account in capital cost computation.

Operating cost

The operating time of 8760 hr/yr is used.

(1) Annual electricity cost for the compressor and humidifiers is

$$= \left(\frac{\$0.042}{\text{kW} \cdot \text{hr}}\right) \left(\frac{1 \text{ kW}}{1.341 \text{ hp}}\right) (W_{\text{comp}} + Q_{\text{humid}}) \left(\$760 \frac{\text{hr}}{\text{yr}}\right)$$

(2) Annual cooling water cost is

$$= \left(\frac{\$C_{\text{w}}}{3.785 \,\text{m}^3}\right) \left(\frac{0.001 \,\text{m}^3}{\text{kg}}\right) \left(\frac{Q_{\text{cool}}}{30}\right) \left(8760 \,\frac{\text{hr}}{\text{yr}}\right)$$

(3) Annual hydrogen cost is

$$= \left(\frac{\$C_{\rm H_2}}{\rm kg}\right) \left(M_{\rm H_2,in} \frac{\rm kmol}{\rm s}\right) \left(3600 \frac{\rm s}{\rm hr}\right) \left(2 \frac{\rm kg}{\rm kmol}\right) \left(8760 \frac{\rm hr}{\rm yr}\right)$$

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